INQUIRY-BASED TEACHING OF FRACTION DIVISION: A CASE STUDY OF PRIMARY SCHOOL MATHEMATICS TEACHERS

Pengajaran Berasaskan Inkuiri Bagi Bahagi Pecahan: Kajian Kes Guru Matematik Sekolah Rendah

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Received: 2/4/2024 Revised: 10/6/2024 Accepted: 10/8/2024 Published: 10/10/2024

DOI: https://doi.org/10.61374/temp10.24

ABSTRACT

This study aims to explore the inquiry-based teaching of fraction division among primary mathematics teachers and to understand their reasoning behind their teaching methods. This paper presents a case study of an experienced teacher, Mr. Ali (pseudonym) who teaches standard six students and employs inquiry-based teaching when teaching fraction division. This study applies a case study design, and the teacher was chosen using the purposive sampling method. Research data was collected through lesson observation, interviews, and document analysis to answer the research questions. The collected data was then qualitatively interpreted and analyzed. The study reveals that the teacher used an inquiry-based method in teaching division of fractions which is anchored on stimulation, hands-on activities and making conclusions. This approach is heavily influenced by Mr. Ali's understanding and beliefs in inquiry-based teaching. He believed that hands-on activities and learning by doing promote fun learning and enhance students' memorization of procedure for fraction division. Additionally, the teacher also believed that inquiry-based teaching is appropriate to be used in teaching the concept of fraction division because it develops higher-order thinking skills among students. Based on these findings, it is recommended that future studies explore the use of inquiry-based teaching of other concepts of fraction to gain a more comprehensive understanding of its application in teaching mathematics.

Keywords: inquiry-based teaching, division of fractions, fractions, case study.

ABSTRAK

Kajian ini bertujuan untuk meneroka pengajaran berasaskan inkuiri bagi bahagi pecahan kalangan guru matematik sekolah rendah dan memahami alasan di sebalik kaedah pengajaran yang mereka laksanakan. Kertas ini membentangkan kajian kes bagi seorang guru yang berpengalaman, iaitu Encik Ali (nama samaran) yang mengajar murid Tahun Enam dan menggunakan pengajaran berasaskan inkuiri semasa mengajar pembahagian pecahan. Kajian ini menggunakan reka bentuk kajian kes, dan guru tersebut dipilih dengan menggunakan kaedah persampelan bertujuan. Data penyelidikan dikumpulkan melalui pemerhatian pengajaran, temu bual, dan analisis dokumen bagi menjawab soalan kajian. Data yang dikumpulkan kemudiannya ditafsir dan dianalisis secara kualitatif. Kajian ini mendapati bahawa guru tersebut menggunakan pengajaran berasaskan inkuiri bagi bahagi pecahan dengan tiga fasa utama, iaitu simulasi, aktiviti hands-on dan membuat kesimpulan. Pendekatan ini sangat dipengaruhi oleh pemahaman dan kepercayaan Encik Ali terhadap pengajaran berasaskan

inkuiri. Beliau percaya bahawa aktiviti hands-on dan pembelajaran melalui pengalaman secara langsung mempromosikan pembelajaran yang menyeronokkan dan meningkatkan ingatan prosedur untuk pembahagian pecahan dalam kalangan murid. Selain itu, guru tersebut juga percaya bahawa pengajaran berasaskan inkuiri sesuai digunakan dalam pengajaran konsep bahagi pecahan kerana ia dapat membina kemahiran berfikir aras tinggi dalam kalangan murid. Berdasarkan dapatan ini, kajian lanjut disarankan untuk meneroka penggunaan pengajaran berasaskan inkuiri bagi konsep pecahan lain bagi mendapatkan pemahaman yang lebih komprehensif tentang penerapannya dalam pengajaran matematik.

Kata Kunci: *pengajaran berasaskan inkuiri, pembahagian pecahan, bahagi, kajian kes.*

INTRODUCTION

Fractions has been widely used in everyday lives and is regarded as one of the important topics in mathematics' curriculum. A deep understanding of the concept of fractions may help students to solve problems in daily activities. In the primary school mathematics curriculum (KSSR), the basic concept of fractions is introduced since Standard One. In Standard Six, students are required to learn fraction division, which is often considered as a challenging topic to learn. Although the procedure for dividing fractions is simple, this topic is conceptually complex and can be challenging for students (Lamon, 2020). They demonstrate a poor grasp of fraction division, and it is still difficult for math teachers to assist that understanding (Ervin, 2017).

Traditionally, fraction division has been taught using algorithm methods such as inverse multiplication to solve fraction division (Ervin, 2017). Students have been taught to solve fraction division without knowing the meaning and the reason behind the inverse multiplication method. Nevertheless, difficulty arises when they are faced with word problems in which fraction division is put into a context to be solved. In such a situation, understanding the meaning behind the concept of fraction division can accelerate students' ability to solve the given problems. It is reflected here that an inappropriate pedagogical teaching method makes it difficult for students to learn and understand the concept of fraction divisions (Lortie-Forgues, Tian, & Siegler, 2015). Teachers should balance teaching algorithmic procedures with engaging their students in understanding the meaning behind mathematical concepts, because they cannot help students learn mathematics meaningfully if they do not fully grasp the core concepts (Siegler & Lortie-Forgues, 2015).

Teachers also must employ an effective pedagogical approach that can actively engage students in building their conceptual grasp of mathematics and mathematical thinking to meet the demands of the current learner-focused views in mathematics education (Boaler, 2016). Accordingly, in the mathematics classroom, opportunities to accomplish this are provided by inquiry-based teaching (Charalambous, 2015). Inquiry-based teaching emphasises understanding the meaning of a concept, learning through hands-on activities and active learning by doing. Manipulatives tools and representations are also used widely in learning activities.

Inquiry-based teaching has also been recognized by past researchers as successful in enhancing students' performance in fractions and increasing their overall interest in learning mathematics. Through this approach, students are expected to solve and make sense of a properly scaffolded series of mathematical tasks given by teachers in groups or individually, which constitutes a type of active learning (Ernst et al., 2017). However, teachers may find it challenging to apply inquiry-based teaching in mathematics lessons due to several challenges it presents for them, as it differs from the traditional approach widely used and normalized by mathematics teachers (Maass & Engeln, 2018). Past research

has also indicated a lack of knowledge on how it has been conducted in a real classroom setting (Gutierez, 2015). Insights from a real classroom setting and the implementation of inquiry-based teaching of fraction division can help teachers understand how it can be effectively implemented. Hence, this study is conducted to explore the teaching of fraction division using the inquiry-based teaching method in a real classroom setting of a primary school mathematics teacher to determine whether the approach can facilitate meaningful teaching opportunities.

The objectives of this study are to explore inquiry-based teaching of fraction division among primary school mathematics teachers and to investigate the reasons behind their instructional approaches. Specifically, the research questions of this study are (a) How do primary school mathematics teachers conduct inquiry-based teaching of fraction division? and (b) Why do the teachers teach the way they do? Answers to these research questions will provide insights into understanding the implementation of inquiry-based teaching of fraction division in a real classroom setting."

LITERATURE REVIEW

Fraction Division

Fraction division poses challenges for both students and teachers (Getenet & Callingham, 2021). Often, it is conceptualized as a straightforward algorithmic process, taught simply as "invert and multiply." However, this topic is theoretically complex and challenging, as its significance requires explanations linked to other areas of mathematics, various representations, or real-world situations. Teachers thus play a crucial role in facilitating students' understanding of fraction division. The way content is presented, and the learning environment can significantly impact students' comprehension of this topic. Therefore, it is suggested that mathematics teachers need a relational understanding of fraction division and should create learning environments with activities that foster students' relational comprehension of the subject (Wahyu et al., 2020).

Previous studies also support the notion that students often encounter difficulties in learning fraction division. These challenges can stem from the nature and complexity of the topic (Lamon, 2020), teachers' mastery of conceptual understanding of fraction division (Copur-Gencturk, 2021), and pedagogical aspects (Getenet & Callingham, 2021). Most past studies have focused on teachers' understanding of fraction division, with few exploring how it should be taught to enhance student comprehension. Therefore, there is a need for further research in this area.

Inquiry-Based Teaching of Mathematics

According to Dorier and Maass (2020), inquiry-based mathematics education is a student-centered approach that encourages students to observe phenomena or situations, seek mathematical solutions, draw diagrams, identify patterns and relationships, make generalizations, and effectively communicate their solutions or findings. Some common terms frequently used to describe inquiry-based teaching are learner-focused, investigation or research-driven, question-driven, communication, reflection, and collaboration (Chapman, 2011). Over the past decade, there has been a growing body of research focusing on inquiry-based teaching of mathematics. This research has shown that inquiry-based teaching activities can be effectively used in mathematics classrooms and provide useful methods and resources for teaching algorithms (Laudano, Tortoriello, & Vincenzi, 2020).

Inquiry-based teaching has been found to have positive impacts on students' learning of mathematics. Research by Karademir and Akman (2019) demonstrates that preschoolers' numerical and operations skills have improved and lasted longer because of inquiry-based teaching activities. Furthermore, inquiry-based teaching has been identified to facilitate a more enjoyable and active learning environment, thus increasing students' motivation in learning mathematics (Karademir &

Akman, 2021). Additionally, the approach helps develop higher-order and critical thinking skills among students (Kandil & Işıksal-Bostan, 2019; Rooney, 2009).

Many past studies on inquiry-based teaching of mathematics have focused on the effectiveness and impacts of the approach on students' learning (Divrik, Pilten, & Tas, 2020; Hastuti et al., 2020). However, very few studies have focused on how it can be implemented in a real classroom setting, particularly in the teaching of fraction division. Hence, this research is carried out to explore inquiry-based teaching of fraction division among primary school mathematics teachers with the goal of outlining specific methods for its implementation.

METHODOLOGY

This study employs a case study design, and the participants consist of five primary school mathematics teachers who were purposively sampled based on predetermined criteria to obtain quality and comprehensive data. Specifically, the selection criteria required that the teachers must have at least five years of experience in teaching primary school mathematics. This is because the teachers will already possess well-developed knowledge and skills in teaching (Hiebert & Carpenter, 1992). Additionally, the teachers must have attended professional development courses in inquiry-based teaching of mathematics organized by the Ministry of Education Malaysia and frequently apply inquiry-based teaching of mathematics.

This paper only focuses on one participant, representative of one case study. By focusing on a single case study, the research can delve deeply into the specific instructional strategies and their impact on students' understanding of fraction division. This depth allows for a comprehensive analysis of the nuances in teaching practices and student responses. The research data was subsequently collected through lesson observations, interviews, and document analysis. To ensure the reliability and validity of the study a triangulation of data collection methods was employed. Lesson observations captured real-time instructional practices and student interactions, while semi-structured interviews with the teachers provided in-depth insights into their experiences and perspectives. Document analysis of lesson plans, instructional materials, and student work offered tangible evidence of teaching strategies and learning outcomes. The qualitative data was analyzed using qualitative content analysis methods (Mayring, 2015).

FINDINGS

The findings of this study provide answers to the research questions; i) "How do primary school mathematics teachers conduct inquiry-based teaching of fraction division?" and ii) "Why do the teachers teach the way they do?". This paper presents one case study of an experienced mathematics teacher, Mr. Ali, who teaches fraction division for a class of 40 students, Standard 6 Cempaka. The time allocated for the lesson is one hour.

Mr. Ali (Pseudonym)

Mr. Ali, an experienced mathematics teacher, has dedicated 25 years to teaching primary school mathematics. His expertise extends to being a contributing writer for modules on inquiry-based teaching of mathematics, published by the Ministry of Education Malaysia (MOE). Recognized as a master trainer by the MOE in inquiry-based teaching of mathematics for teachers nationwide, Mr. Ali implements inquiry-based teaching methodologies, including in the teaching of fraction division, within his mathematics classroom. The following details how he employs inquiry-based teaching methods to teach fraction division and elucidates the rationale behind his instructional approach

(a) How does the teacher teach fraction division using inquiry-based teaching?

Mr. Ali's approach in using inquiry-based teaching of fraction division is discussed in three themes that emerged from the data analysis. The themes are (i) stimulation, (ii) hands-on activities, and (iii) making conclusions.

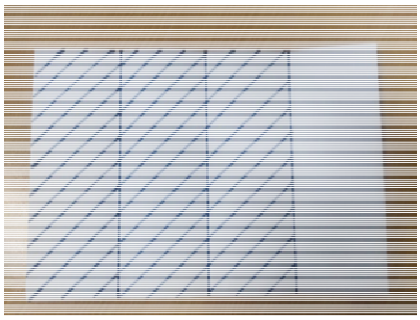
(i) Stimulation

The stimulation phase emphasizes the significance of employing real-life scenarios, visual aids, and questioning techniques to captivate students' curiosity and interest in the subject. Mr. Ali utilized a problem associated with a real-life situation as the focal point during this phase and displayed the problem on the whiteboard. An illustrative example of the problem is outlined below:

> "Fatimah had ¾ cake left from a birthday party. She wanted to divide the cake into half and give one half to Suzy. What is the fraction of the cake that Suzy gets?"

Mr. Ali utilized a visual aid (Figure 1), a piece of paper depicting a ¾ cake, which he then presented to the class.

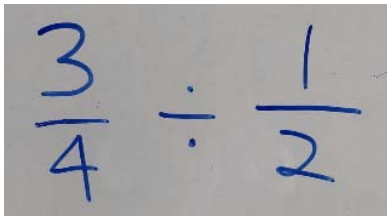
Figure 1
Picture of ¾ 'cake'



He read the problem aloud and inquired of the students, "What fraction of the cake did Suzy receive?" Despite his prompt, the students remained silent. Undeterred, he persisted, asking, "How can we tackle this problem?" However, the students expressed their uncertainty, informing Mr. Ali, "We don't know, teacher." Responding to their uncertainty, Mr. Ali attempted to guide them by prompting them on how to initiate problem-solving. "Alright, typically, what do we do when approaching word problems? We aim to formulate mathematical expressions for the problem. Please jot down your response on your mini whiteboard."

The students proceeded to write their answers on their mini whiteboards, while Mr. Ali circulated around the class to inspect their responses. By providing mini whiteboards, Mr. Ali facilitated the students' ability to visualize their thought processes. It was noted that the majority of students composed a mathematical expression, as depicted in Figure 2 below.

Figure 2
Student's answer



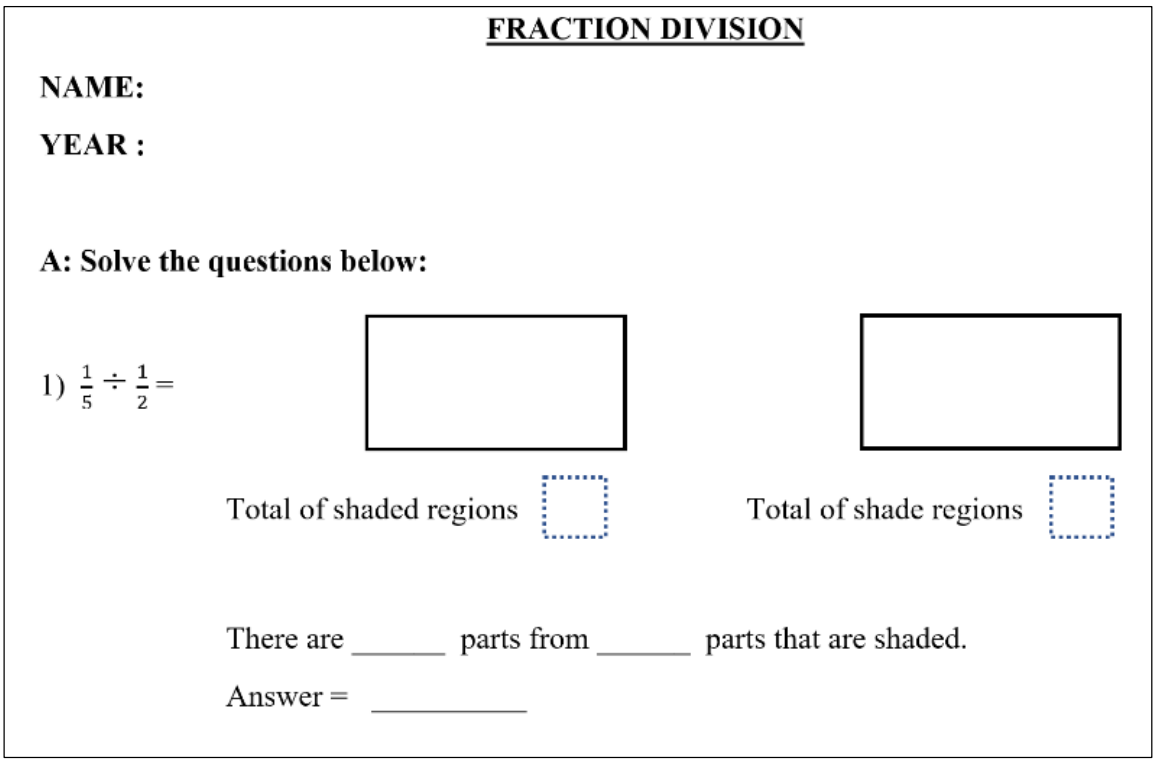
Next, Mr. Ali posed a question to the students: "Why did you use the division symbol?" to which the students responded, "Because we need to divide the cake, so it's division." Mr. Ali concurred with the students' explanation and proceeded, "Alright, let's delve into how to divide fractions by fractions."

During this stimulation phase, Mr. Ali did not aim for students to arrive at the correct answer to the given problem; instead, he employed the problem as a means to engage the students and connect it back to the lesson. Subsequently, the lesson progressed to the next phase, characterized by a hands-on activity.

(ii) Hands-on activities

The hands-on activities observed in this study involved sketching and drawing diagrams to help students grasp the concept of fraction division and to solve fraction division problems using visual aids. Students were seated in groups and were prompted to engage in discussions with their group members while attempting to solve the problems provided in the worksheet. The worksheet contained five fraction division questions, each accompanied by an empty diagram for students to sketch. An example of one of the questions from the worksheet is illustrated in Figure 3.

Figure 3
Question 1 in the worksheet



For the initial question on the worksheet, Mr. Ali provided guidance to the students by illustrating diagrams on the whiteboard at the front of the class. The steps for sketching and drawing diagrams are outlined in Figure 4. Mr. Ali initiated by posing the question, "How many is 1/5 inside of 1/2?" He then

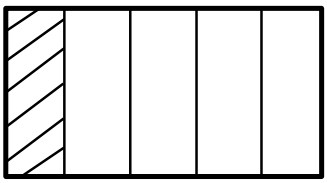
proceeded to draw diagrams on the whiteboard to elucidate the problem, as depicted in Figure 4. An exemplar of a student's response is presented in Figure 5.

Figure 4

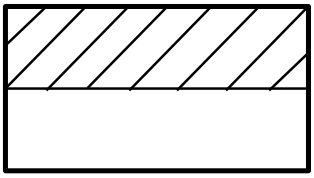
Steps for drawing and sketching diagrams to solve $\frac{1}{5} \div \frac{1}{2}$.

Teacher's question: How many is $\frac{1}{5}$ in $\frac{1}{2}$?

a) Sketch $\frac{1}{5}$ and $\frac{1}{2}$ in different boxes.



$\frac{1}{5}$

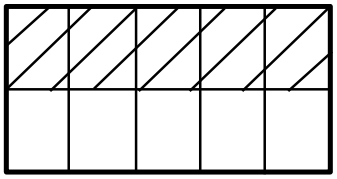


$\frac{1}{2}$

b) Equalize the number of parts in each box.

For the box showing $\frac{1}{5}$, draw one horizontal line to make it like the box showing $\frac{1}{2}$

For the box showing $\frac{1}{2}$, draw five vertical lines like make is like the box showing $\frac{1}{5}$



c) Count the number of shaded regions in diagram $\frac{1}{5}$ = 2 shaded regions

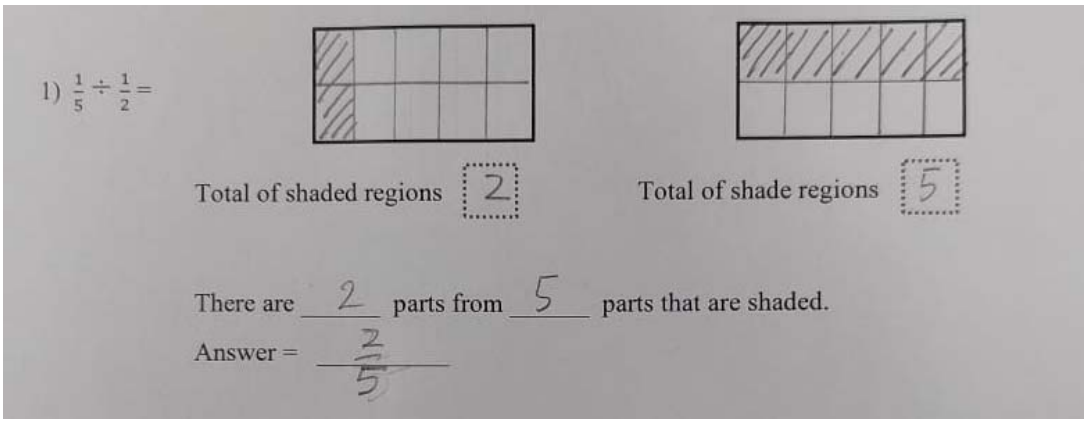
d) Count the number of shaded regions in diagram $\frac{1}{2}$ = 5 shaded regions

e) There are 2 parts from 5 parts of the shaded regions

f) Answer: $\frac{1}{5} \div \frac{1}{2} = \frac{2}{5}$

Figure 5

Example of a student's answer for question 1.

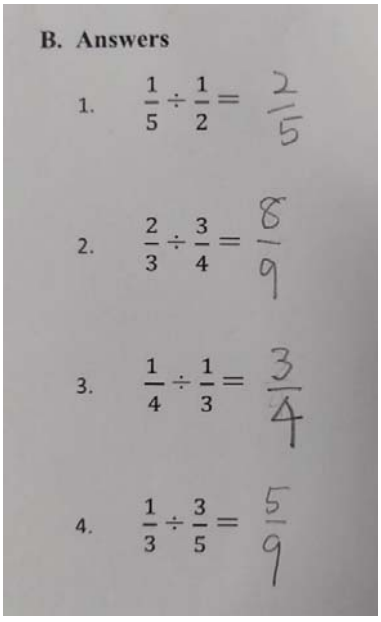


(iii) Making conclusion.

Making conclusions highlights students' capacity to articulate the procedure for dividing fractions based on the answers derived during the hands-on activity. They compiled all the answers for each question, as depicted in Figure 6. Subsequently, they were tasked with identifying the pattern to obtain the solutions.

Figure 6

List of answers



Mr. Ali utilized the following questions to prompt the students to formulate a conclusion about fraction division problems:

a. Can you discern a pattern in deriving the answers for the given questions?

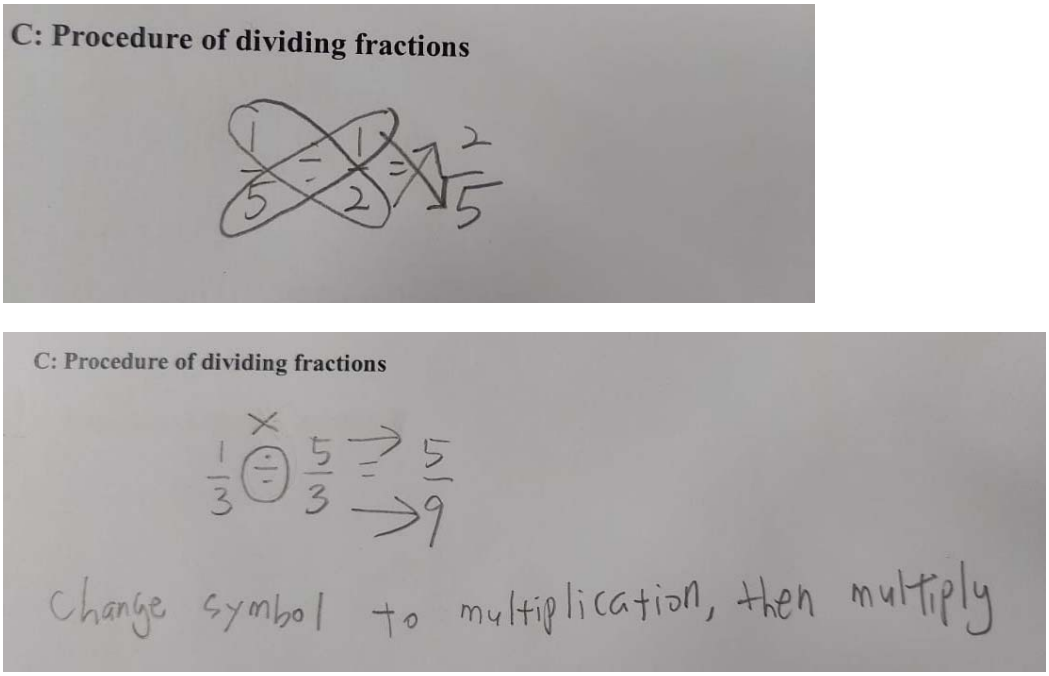
b. Do you grasp the steps required to obtain the answers for all the questions we just worked on?

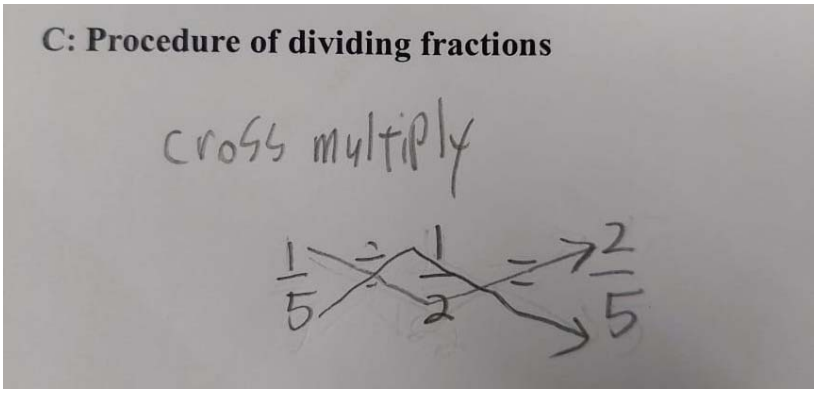
c. Are there alternative methods to obtain the answers to the questions we addressed, besides sketching diagrams?

Following this, some students responded, "We can use cross multiplication, teacher!" In response, Mr Ali invited one of the students to step forward and illustrate their answer. The student elaborated that cross multiplication can also be employed for fraction division, wherein the numerator of the first fraction is multiplied by the denominator of the second fraction, and vice versa (refer to Figure 7). Additionally, there were students who proposed that the division symbol could be replaced with the multiplication symbol. They suggested completing the multiplication by multiplying the numerator of the first fraction by the numerator of the second fraction, and then the denominator of the first fraction by the denominator of the second fraction (refer to Figure 7). Other students advocated for continuing to use the division symbol. Consequently, the students drew the following conclusions, as depicted in Figure 7, regarding solving fraction divisions.

Figure 7

Three examples of students' answers on fraction division procedure





(b) Why do the teachers teach the way they do?

From the interviews, it is found that the teachers' approach to teaching fraction division using inquiry-based was influenced by their knowledge and beliefs on the effectiveness of this approach. The teachers believe that inquiry-based teaching is beneficial for promoting conceptual understanding, developing thinking skills, and making learning fun for students.

Mr. Ali has consistently favored teaching fraction division using the inquiry-based teaching method ever since he received training in this approach. He is convinced that inquiry-based teaching is the ideal method for teaching fraction division, as it effectively promotes conceptual understanding among students, nurtures critical thinking skills, and enhances the enjoyment of learning. This assertion was confirmed by Mr. Ali during the interviews, as illustrated by the following excerpt:

> *"Ever since I was introduced to this method, I have employed it to teach fraction division, and I believe my students are demonstrating a keen interest in learning mathematics, particularly fraction division. This approach promotes conceptual understanding, encourages critical thinking among my students, and actively involves them in the learning process..."*

Mr. Ali believes that to start the lesson, the teacher must be able to get the students' attention and the simulation phase will provide the students with initial ideas on what they will learn during the lesson. This increases students' interest in learning and engages them throughout the lesson. Mr. Ali confirmed this during interviews as shown by the following interview excerpt:

> *"For the induction phase, it's important to capture students' attention before the main lesson begins. One effective approach is to present students with problematic situations related to dividing fractions, allowing them to grasp the topic's essence before delving into the lesson."*

Mr. Ali believes that hands-on activities offer students the opportunity to construct knowledge from their own experiences, fostering idea generation, problem-solving, and creating an enjoyable learning environment.

> *"Hands-on activities play a crucial role in inquiry-based teaching by sparking students' curiosity. Through these activities, students construct knowledge from their own experiences, fostering the generation of ideas and solutions. Without hands-on engagement, information transmission becomes one-sided, lacking student involvement. Active participation in hands-on learning not only enhances understanding but also makes the learning process more enjoyable for students."*

Mr. Ali advocates for the effectiveness of group work as a method to foster critical thinking and problem-solving skills in the classroom. By engaging in collaborative problem-solving activities, students learn to work as a team to tackle challenges, particularly those related to fraction division. This approach not only enhances their mathematical abilities but also cultivates communication and teamwork skills, both of which are essential components of critical thinking.

> *"This approach to inquiry prompts students to engage in critical thinking. When conducting hands-on activities in groups, students are encouraged to discuss and collaboratively solve problems. These discussions not only foster the development of communication skills but also stimulate critical thinking as students encounter and reconcile conflicting ideas."*

DISCUSSION AND SUMMARY

The inquiry-based teaching of fraction division by primary school mathematics teachers was explored by using case study and data were collected through lesson observations, interviews, and document analysis to answer the research questions.

The three themes that emerged from the data are stimulation, hands-on activities, and making conclusions. These are consistent with the principles of inquiry-based teaching which emphasize active learning, exploration, and reflection.

The first theme, stimulation, highlights the importance of using real-life examples, visual aids, and thought-provoking questions to engage students' interest and curiosity in the topic. This helps students see the relevance of the topic to their own real-life experience, thus promoting their motivation to learn. This theme aligns with research by Brown (2017) finds that the use of real-life examples, visual aids, and thought-provoking questions enhances students' interest in the topic, making the learning experience more stimulating and relevant.

The second theme, hands-on activities, emphasizes the importance of providing students with concrete and tangible experience on the concept of fraction division. Hands-on activities in math education have been shown to promote students' problem-solving skills, reasoning skills, and conceptual understanding of mathematics. In the learning fraction division, providing students with this helps them to develop much deeper understanding and more meaningful engagement with the learning material (Schwichow et al., 2016).

Finally, the third theme, making conclusions, emphasizes the importance of allowing students to reflect on what they have learned and draw connections between different concepts. According to Costes-Onishi, Baildon and Aghazadeh (2020), making conclusions is a critical aspect of inquiry-based learning, as it promotes students' metacognitive skills and their ability to transfer knowledge to new situations. By reflecting on what they have learned and drawing connections between different concepts, students can better understand the relevance of the topic to their lives and apply their learning in real-world contexts.

However, according to Ryan and St-Laurent (2016), although inquiry-based learning can be effective in promoting student learning outcomes, it is not a one-size-fits-all approach. Some students may not be highly motivated by real-life examples or hands-on activities and may prefer more traditional forms of instruction when learning mathematics. Hence, it is important for teachers to recognize and accommodate to their students' different learning preferences and styles when implementing inquiry-based teaching.

This research also finds that the teachers' approach to teaching fraction division using inquiry-based teaching is influenced by their knowledge and beliefs about the effectiveness of this approach. The teachers believe that inquiry-based teaching is beneficial for promoting conceptual understanding, developing thinking skills, and making learning fun for students. This aligns with previous research that has shown the benefits of inquiry-based teaching in promoting student engagement, motivation, and understanding of mathematical concepts. For example, a study by Tella and Sulaimon (2022) finds that inquiry-based teaching can improve student engagement and understanding of mathematical concepts in fractions. Similarly, a study by Duran and Dökme (2016) stipulates that inquiry-based teaching can promote critical thinking and problem-solving skills in students.

Overall, the findings of this study highlight the importance of using inquiry-based teaching to promote conceptual understanding and thinking skills in mathematics education, specifically in the teaching of fraction divisions. Nevertheless, it remains important for teachers to consider the individual needs of students and contextual factors when deciding on an instructional approach.

In summary, this study concludes that the teachers' approach to teaching fraction division using inquiry-based teaching is influenced by their knowledge and beliefs about the effectiveness of this approach. The themes of stimulation, hands-on activities, and making conclusions emerged from the data and emphasize the importance of active learning, exploration, and reflection in promoting student engagement and understanding. These findings are consistent with previous research and highlight the importance of considering individual needs and contextual factors in instructional decision-making.

Acknowledgements

I extend special thanks to Dr. Suzieleez Syrene Abdul Rahim and Dr. Edy Hafizan Mohd Shahali for their invaluable guidance as the supervisor and co-supervisor respectively.

Disclosure Statement

No potential conflict of interest was reported by the author.

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